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ANALYSIS OF PRIVATIZATION IN STACKELBERG MIXED OLIGOPOLY

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Abstract

Mixed oligopoly with one welfare-maximizing public and several profit-maximizing private firms exists in many economies. De Fraja and Delbono (1989) have analysed mixed oligopoly taking into account how the public firm behaves vis-à-vis the private firms on the basis of a linear market demand function and symmetric firms. They have found that the social welfare is greater in Stackelberg mixed oligopoly where the public firm acts as a leader than in Cournot mixed oligopoly where all firms simultaneously determine their outputs. A partial public firm tries to maximize the weighted average of the social welfare and its profits. Under some conditions, partial privatization of a public firm leads to greater social welfare than Cournot mixed oligopoly where the public firm is fully public (see Matsumura (1998) for duopoly and Okuguchi (2012) for oligopoly). In this paper we will prove that neither partial nor full privatization of a public firm is optimal in a general Stackelberg mixed oligopoly where the public firm acts as a leader and all private firms as followers.

Key words: public firm, Stackelberg mixed oligopoly, privatization

1. INTRODUCTION

The existence of mixed oligopoly where a public firm and private ones coexist have been observed and analyzed first by Merrill and Schneider (1966), and later by Harris and Wiens (1980), Beato and Mas-Colell (1982), Boes (1986,1991),and Creamer *et al* (1987) among others. De Fraja and Delbono (1989) (see also De Fraja and Delbono,1990) have compared the welfare of mixed oligopoly consisting of one fully public firm and several symmetric private firms for four possible cases distinguished by the public firm's behavior in relationship to all private firms. They have assumed a linear market demand function for an identical good produced by all firms and the same quadratic cost function for all firms ,and found, among other things, that the social welfare is greater in Stackelberg mixed oligopoly where the public firm acts as a leader and all private firms behave as followers than in Cournot (or Nash) mixed oligopoly where the public and private firms simultaneously choose their outputs. Some more recent contributions to Stackelberg mixed oligopoly, especially in relationship with the effects of subsidies to firms , include Poyago-Theotoky(2001), Myles (2002), Cornes and Sepahvand (2003), Fjell and Heywood (2004,2007) and Zikos (2007).Myles (2002) adopts most general approach among them and assumes away a linear market demand function and quadratic cost funcions.However, he assumes identical cost functions for all firms,including the public one.

A partial public firm whose manager maximizes the weighted average of the social welfare and its profits have widely been observed in many economies. Under certain conditions, partial privatization of a public firm results in greater social welfare than in Cournot mixed oligopoly where the public firm is under full control of the government. This has been shown by Matsumura(1998) for duopoly and by Okuguchi(2012) for oligopoly. In this paper we will systematically analyze under very general conditions on the market demand and firms' cost functions whether partial or full privatization of a public firm in Stackelberg mixed oligopoly with a public firm as a leader isoptimal or not . We will find that neither partial nor full privatization of the public firm is optimal.This non-optimality result coincides with the one earlier obtained by De Fraja and Delbono(1989) for their simple case of a linear market demand function and the identical cost function for all firms.We will be able to derive our result remarkably easily by taking the public firm's rest of the industry output as an analytical strategic variable, as it is uniquely related to its own output as shown below.

Before concluding this introductory section, we would like to point out the pioneering paper on Stackelberg oligopoly with only private firms by Sherali *et al.*(1983) from the algorithmic point of view of computing the Stackelberg equilibrium.

2. MODEL AND ANALYSİS

Let there be one public firm(firm *0*) and *n* asymmetric profit-maximizing private firms. Let π_i , $i = 0,1,...,n$,be firm *i*'s profits, *W* be the social welfare as the sum of firm's profits and the consumers' surplus, and $U_0 \equiv \alpha W + (1-\alpha)\pi_0$ be the partial public firm's objective function, where the parameter $\alpha \in [0,1]$ is the weight the government attaches to the social welfare .

If $\alpha = 0$,the public firm becomes a private profit-maximizing firm and if $\alpha = 1$, it is a fully public firm.Furthermore,let $X \equiv \sum_{i=0}^{n} x_i$ be the industry total output, where x_i is firm *i*'s output, $p = p(X)$ the inverse market demand function, where *p* is the price of a homogeneous good of the industry and *p*'<0 for *X* such that $p > 0$, and $C_i(x_i)$ be firm *i*'s cost function. Then, by definition of the social welfare W ,

$$W \equiv \int_0^X p(x)dx - \Sigma_{i=0}^{n} C_i(x_i). \tag{1}$$

The firm *i*'s profit function is

$$\pi_i = x_i p(\sum_{j-0}^{n} x_j) - C_i(x_i), i = 0,1,2,...,n. \tag{2}$$

We now formulate Stackelberg mixed oligopoly with a partial public firm as a leader and all private firms as followers. The public firm's objective function is U_0 defined above and equals to the weighted average of the social welfare and its profits. We rewrite the firm's profit function as

$$\pi_i = x_i p(x_0 + X_{-0}) - C_i(x_i), i = 0,1,...,n, \tag{3}$$

where an analytical strategic variable $X_{-0} \equiv \sum_{i=1}^{n} x_i$ is the rest of the industry output for the public firm. De Fraja and Delbono (1989) have assumed a linear market demand function for the good and an identical quadratic cost function for all firms, while Beato and Mas-Colell (1982) have used a linear market demand function and general cost functions for mixed duopolists. We will, however, assume general demand and cost functions which are assumed to satisfy:

Assumption 1: $C_i^{'} > 0, i = 0,1,...,n, p^{'} < C_i^{''}, i = 1,2,...,n$.

Assumption 2: $p' + x_i p'' <,0, i = 1,2,...,n.$

Now let the leader's output x_0 be given.Then private firm *i* maximizes its profit with respect to its own output on the basis of the Cournot behavioristic assumption regarding its rival's outputs. Hence, its first order condition for profit maximization.

$$\frac{\partial \pi_i}{\partial x_i} = p(x_0 + X_{-0}) + x_i p'(x_0 + X_{-0}) - C_i^{'}(x_i) = 0, i = 1,2,...,n, \quad (4)$$

where we have assumed an interior maximum.The Assumption 2 implies that any two private firm's outputs are strategic substitutes each other. Note that the second order condition holds under Assumptions 1 and 2. We note in passing that the equation (4) shows that the private firms are playing an aggregative game among themselves(see Okuguchi and Yamazaki(2014)).

Solving (4) with respect to x_i as a function of $x_0 + X_{-0}$, we have

$$x_i \equiv \varphi^i (x_0 + X_{-0}), i = 1,2,...,n, \quad (5)$$

where

$$\frac{d\varphi^i}{dX} = \frac{\partial \varphi^i}{\partial x_0} = \frac{\partial \varphi^i}{\partial X_{-0}} = -\frac{p' + x_i p''}{p' - C_i^{''}} < 0, i = 1,2,...,n. \quad (6)$$

Note that (5) is not the reaction function in the traditional sense of the word since φ^i contains x_i as one of its arguments because of $X_{-0} \equiv \sum_{i=1}^{n} x_i$.

By definition ,the rest of the industry output for the public firm is

$$X_{-0} = \sum_{i=1}^{n} \varphi^i (x_0 + X_{-0}) \equiv \varphi^{-0}(x_0 + X_{-0}). \quad (7)$$

Solving (7) with respect to the leader' output, we have

$$x_0 \equiv \Psi(X_{-0}), \quad (8)$$

where in view of (6),

$$\Psi'(X_{-0}) = \frac{dx_0}{dX_{-0}} = \frac{1}{\sum_{i=1}^{n} \frac{d\varphi^i}{dX}} - 1 < -1. \quad (9)$$

We can give an diagrammatic derivation of (8) as follows.If the public firm's output is x_0' ,the solution of (7) corresponds to the intersection E' of a downward-sloping curve for $\varphi^{-0}(x_0' + X_{-0})$ and the 45 degree line originating from the origin as shown in the Figure1 below.

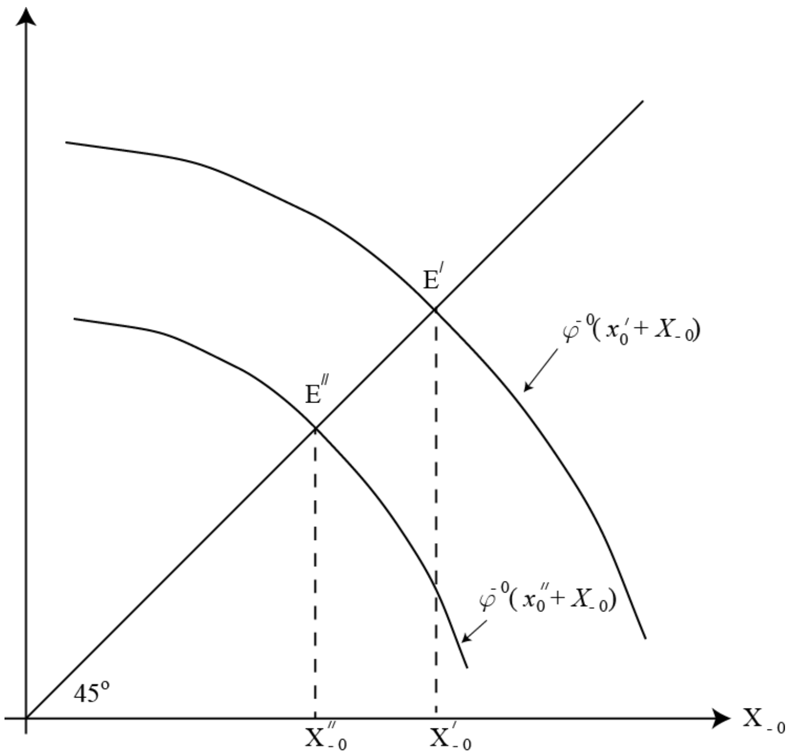

Figure1The solution of (7)

If the public firm's output increases to x_0'' ,the curve shift downwards,and the new intersection becomes E'',hence X''_{-0} <X'_{-0} for x_0' <x_0'' .

The public firm's objective function now reads

$$U_0 = \alpha\left\{\int_0^{x_0+X_{-0}} p(x)dx - \sum_{i=0}^{n} C_i(x_i)\right\} + (1-\alpha)\{x_0 p(x_0 + X_{-0}) - C_0(x_0)\}$$

$$= \alpha\left\{\int_0^{X_{-0}+\Psi(X_{-0})} p(x)dx - C_0(\Psi(X_{-0})) - \sum_{i=1}^{n} C_i(\varphi^i(X_{-0} + \Psi(X_{-0})))\right\} +$$

$$(1-\alpha)\{\Psi(X_{-0})p(X_{-0} + \Psi(X_{-0})) - C_0(\Psi(x_{-0}))\} \quad , \qquad (10)$$

where we have taken into account (5) and (8).Given α ,the manager of the public firm maximizes its objective function U_0 with respect to its outpu x_0 ,that is,with respect to its rest of the industry output X_{-0} in light of (8).The first order condition for maximization of U_0 with respect to X_{-0} is rewritten as

$$V(X_{-0},\alpha) \equiv \alpha A(X_{-0}) + (1-\alpha)B(X_{-0}) = 0 \tag{11}$$

where

$$A(X_{-0}) \equiv \left\{ p(\beta(X_{-0})) - \sum_{i=1}^{n} C_i^{'}(\beta(X_{-0}))\varphi^{i'}(\beta(X_{-0})) \right\} \times$$
$$(1+\Psi'(X_{-0})) \ - C_0^{'}(\Psi(X_{-0}))\Psi'(X_{-0}),$$
$$B(X_{-0}) \equiv p(\beta(X_{-0}))\Psi'(X_{-0}) + \Psi(X_{-0})p'(X_{-0})(1+\Psi'(X_{-0}))$$
$$-C_0^{'}(\Psi(X_{-0}))\Psi'(X_{-0}),$$

$$\beta(X_{-0}) \equiv X_{-0} + \Psi(X_{-0}).$$

We introduce here the following second order condition.

Assumption 3: $\dfrac{\partial^2 U_0}{\partial X_{-0}^2} < 0$.

In order to show the validity of this assumption,consider the following case in which the market demand function is linear, the public firm's cost function is quadratic and all private firm's cost functions are linear and identical.

$$P = a - bX, C_0 = {c_0 x_0^2}/{2}, C_i(x_i) = cx_i, i = 1,2,...,n. \tag{12}$$

A simple calculation yields

$$A(X_{-0}) = {-b\{b + c_0(n+1)^2\}X_{-0} + n(n+1)\{c_0(a-c) - bc\}}/{bn^2} \gtrless 0, \tag{13}$$

$$B(X_{-0}) = {-b(n+1)\{2b + c_0(n+1)\}X_{-0} + C}/{bn^2} \gtrless 0, \tag{14}$$

$$A^{'}(X_{-0}) = {-\{b + c_0(n+1)^2\}}/{n^2} < 0, \tag{15}$$

$$B^{'}(X_{-0}) = -(n+1)\{2b + c_0(n+1)\}\Big/ n^2 < 0\,, \tag{16}$$

$$C \equiv n\{ab + c_0(a-c)(n+1) - (n+2)bc\}. \tag{17}$$

In view of Inequaliti s (15) and (16),we know tha the second order condition is satisfied for the model given by (12).

Under the Assumption 3,we solve (11) with respect to X_{-0} as a function of the parameter α .

$$X_{-0} \equiv X_{-0}(\alpha)\,, \tag{18}$$

where we have in view of the Assumption 3

$$\frac{dX_{-0}}{d\alpha} = \frac{B}{\alpha \dfrac{\partial^2 U_0}{\partial X_{-0}^2}} \begin{matrix} < \\ > \end{matrix} 0 \quad \text{according as} \quad B \begin{matrix} > \\ < \end{matrix} 0. \tag{19}$$

Furthermore, we have in light of (9) and (18),

$$\frac{dX}{d\alpha} = (1+\Psi')\frac{dX_{-0}}{d\alpha} \begin{matrix} > \\ < \end{matrix} 0 \text{ according as } B \begin{matrix} > \\ < \end{matrix} 0. \tag{20}$$

Since the government’s objective function is

$$W(\alpha) = \int_0^{X(\alpha)} p(x)dx - C_0(\Psi(X_{-0})) - \sum_{i=1}^{n} C_i(\varphi^i(X(\alpha)))\,, \tag{21}$$

differentiation of it with respect to α yields

$$\frac{dW}{d\alpha} = \left\{(p - \sum_{i=1}^{n} C_i^{'}\varphi^{i^{'}})(1+\Psi') - C_0^{'}\Psi'\right\}\frac{dX_{-0}}{d\alpha}$$

$$= \frac{(\alpha-1)B}{\alpha}\frac{dX_{-0}}{d\alpha} > 0 \ \text{ for } \ B \neq 0\,. \tag{22}$$

This proves that if $B \neq 0$, the social welfare is maximized for $\alpha = 1$, that is, the public firm should be neither partially nor fully privatized.

3. CONCLUSION

In this paper we have analyzed whether partial or full privatization of a public firm coexisting with several profit-maximizing private firms is optimal in the sense of social welfare maximization.We have given the role of leadership to the public firm which is assumed to be maximizing its objective function as the weighted sum of the social welfareand its profits, and the followership role to all private firms. We have found without assuming a linear market demand function and quadratic cost functions for all firms that neither parial nor full privatization of the public firm is optimal. This finding is in sharp contrast with that of the optimality of partial privatization of the public firm in Cournot mixed oligopoly where all firms are assumed to act as Cournot oligopolists.

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